Funnel Flight Controller for a Simulation Model and Attitude Control System
漏斗飞行控制器用于仿真模型和姿态控制系统

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Abstract - In the recent years modeling has become an important tool for predicting the behavior and the design of dynamical systems resulting in a reduced need for numerous and often expensive testing. Especially if the system under investigation cannot be tested completely under accessible test conditions on ground, the development of reliable simulation tools is mandatory for its design.

Within this paper, the development of an advanced simulation tool is presented which is able to predict the behavior of a so-called Attitude Control System (ACS) which is used to adjust the attitude of a missile to agility requirements commanded by the onboard guidance, navigation and control device. Here the focus is laid on the internal dynamics of such an ACS.

In general, an ACS can be regarded as a small scale rocket engine with the capability of providing thrust at certain points of the operational activity. This is usually realized by a set of circumferentially mounted "mini" motors which can be triggered individually while the missile performs a rotational movement along its longitudinal axis or by a continuous burning (solid propellant) motor which feeds a throttleable Cartesian nozzle configuration.

The advantage of such a system lies in the fact that it provides a continuous operation capability. The thrust direction can be controlled by a simultaneous modification of the four nozzle throats, usually realized by a hot gas valve system consisting of so-called pintles which can be shifted (either two coupled or four individually) backwards and forth to increase or reduce the respective nozzle throat area.

For the current investigation an ACS is simulated which consists of a gas generator that produces a hot gas which is led into a pair of throttleable nozzles. The nozzles are aligned along a straight line and throttled by a coupled pintle system. The pintles connected via a metallic rod are shifted backwards and forth to change the respective nozzle throats such that with closing one throat the other will be opened. The rod is connected via a lever with an actuator.

Such a configuration allows very fast thrust modifications by side thrust change and due to this a very fast position change of the missile depending of the location of the ACS with respect to the missile’s center of gravity.

This paper comprises the simulation model and application for such an ACS containing brief descriptions of the flight control algorithms and a mathematical simulation model. It investigates the performance prediction for the ACS.

For all models a funnel controller method is used as an automatic control tool. It is shown how the funnel controller is used in a simulation and for a real test facility trial aiming at a fast thrust variation.

The theoretical funnel control and experimental results of a laboratory rotatory mechanical system from [SIAM2013] are combined and extended to speed funnel control with nonlinear disturbances and elastic deformation without active damping where only thrust measurement is available. The extended innovative funnel control concept is applied to this highly reactive ACS and guarantees extremely fast thrust tracking with a required transient accuracy in the low millisecond range to achieve maximum performance. The knowledge of all system parameters, the nonlinear mass flow link and the actuator interaction is not required.

Keywords - Funnel control, Attitude Control System, nonlinear systems, transient behavior, solid propellant, gel propellant, simulation model, propellant burnback, Matlab

I. INTRODUCTION

This paper investigates the accuracy of a performance and design model in simulating the thrust generation, thrust direction, combustion pressure behavior, pintle movement, actuator behavior, propellant choice and propellant grain geometry (for solid propellant) etc. of an actively controlled ACS complemented by a methodology to properly model the
system by a precise mathematical description. The basic physical equations of the dynamic can be found in [ROMA2009], [TAM2004], [WILE2001].

The simulation of the performance of an actively controlled ACS is a combination of all the characteristics of the system (e.g. mass properties, performance requirements, propellant burning and combustion gas supply, geometrical configurations), environmental conditions and actuator behavior. The tracking of a changing thrust profile (thrust direction and thrust level) requires a permanent adjustment of the commanded pintle position of the ACS (i.e. actuator command in voltage) in order to compensate the differences between the real measured and the nominal thrust due to manufacturing, measurement, atmospheric modeling errors and due to material failures during operation of the system (e.g. insulation fracture and/or particle deposition in the nozzle throat area).

A detailed simulation model enables engineers to investigate the dynamics and performance during each design phase. In the current model, the significant parameters (e.g. combustion pressure, nozzle exit pressure, thrust etc.) and component behaviors (e.g. actuator behavior model) are determined by different methods – experimentally (e.g. nozzle erosion), CFD analysis (e.g. nozzle design and performance) as well as semi-empirical (e.g. actuator behavior model) and analytical methods (e.g. propellant burning, pressure and thrust generation). The first group obtains data from test facility trials and flight tests. The CFD methods solve a set of fundamental equations for fluid domains – divided into discrete cells – to simulate flow fields and forces (e.g. internal forces and pressure along the boundary layer). At last, the semi empirical and analytical methods calculate the behavior via analytical formulas and an existent empirical database. The last method is the quickest way to determine or approximate the performance of a new component of the dynamical system or the missile/system performance and is thus important during development.

In order to ensure the correct functioning of the system in all phases, a robust and reliable control algorithm has been developed. Its effectiveness has been proven in numerous ground trials. A brief overview of the controller is given in the section of the performance funnel description in this paper. The interested reader is referred to [UVIL2010] for a detailed description.

II. ATTITUDE CONTROL SYSTEM: FUNCTION AND REALIZATION

ACS’s use (impulse) force to control the attitude (pitch, yaw and roll) of a vehicle flying inside or outside the atmosphere. Vehicles using an ACS system are upper stages of space launchers, missiles or missile defense interceptors designed to intercept (ballistic) missiles and their effectors.

Missiles with an ACS system (see Figure 1) are Lockheed-Martin’s PAC 3 ground based air defense missile incorporating an array of many small solid impulse thrusters and MBDA’s CAMM ship-based air defense missile with a continuously operating solid gas generator to turn the missile after vertical launch and point it towards the correct direction. Well-known examples of missile defense systems are the Terminal High-Altitude Area Defense System (THAAD), which uses liquid propulsion to maneuver an endo-atmospheric vehicle and the longer range SM-3 missile, which uses solid propellant propulsion to maneuver an exo-atmospheric kill vehicle.

In space applications, ACS blocks, often designated as “Vernier Motors”, can be used to control the attitude of the launch vehicle without the need to integrate a movable nozzle into the solid propellant rocket motor or to move the liquid propellant rocket motor. Well-known applications are the Vernier motors of the Sojuz launcher.

Currently, liquid and solid propellant ACS are used in operational vehicles. The liquid ACS of the Ground Based Interceptor (GBI) uses individual ACS thrusters from space propulsion technology. These thrusters operate intermittently and use hydrazine or a derivative as propellant. The solid ACS of the SM3 has a central combustion chamber with supposedly three solid propellant grains that produce a mission-adapted level of gas flow for the ACS along the mission.

For vehicles that fly within the atmosphere, ACS’s are in use to increase the agility of a missile in conditions where the aerodynamic forces do not generate sufficiently high pitch and yaw moments or in order to secure a direct hit. This can be the case immediately after the launch of a missile, when the low velocity does not generate the aerodynamic control force needed for a quick turn or in the final approach to a maneuvering target at high altitude, when the low atmospheric pressure does not support quick control of pitch and yaw. The thrust vector of an ACS creates a lateral force at a distance of the Centre of Gravity (CoG) of the missile. The thrust vector of an ACS creates a lateral force at a distance of the CoG of the missile.

The introduced ACS concept assumes a central and continuously operating gas generator to supply the nozzles with produced gases which create the necessary thrust directions. A hot gas pipe transports the produced gas from the gas generator to the ACS.
The design uses four coupled, controlled nozzles with plenum, pintles and actuators, i.e. the two pintles of one nozzle plane are connected. The system has a quasi-Cartesian configuration with pintles to regulate the nozzle throat and control the thrust direction, i.e. the system has two planes with two nozzles. The actuators are outside of the hot gas. The design assumes coupled nozzles to steer the gas flow through the nozzle pair to reduce complexity and number of actuators. A scheme of the system can be seen in Figure 2.

The actual design assumes that the gas generator supplies the nozzle plenum over a hot gas pipe with thermal insulation and efficiency loss. A complete closure of a nozzle is a harder challenge then a “nearly closed” condition. Therefore, the design concept assumes a “nearly closed” nozzle.

![ACS configuration with four nozzles and actuators](image)

Figure 2: ACS configuration with four nozzles and actuators

Figure 2 shows the basic building blocks of an ACS with the exception of the electric energy and gas supply. In most designs a gas generator supplies the gas that is distributed by the nozzle, pintle blocks according to the thrust demand. The ACS thrusters are located outside the CoG near the periphery. For missile applications, the total time of operation of ACS is typically some seconds depending on the mission profile.

Continuously operating ACS’s, as shown in Figure 2, consist of three main building blocks - namely the combustion gas supply (CGS), the nozzle, pintle, actuator blocks that distribute the gas to the ACS’s nozzles and the control unit that controls the pintle and actuator and, if possible, the CGS system, to convert the signals from the guidance, navigation and control (GNC) into thrust vectors of correct magnitude and direction. In many applications the nozzles and the pintles are one assembly that is very tightly connected with the actuators. The mechanical interface between CGS and nozzles can be described comparatively easily as a flow of hot combustion gas with determined mass flow at a given set of state parameters. This allows to take pintle nozzles with actuators and CGS as separate mechanical entities. More complicated are the interfaces of the control unit with both pintle nozzle actuator and CGS if the CGS can produce a variable mass flow (e.g. gelled propellant gas generator or TDR with solid propellant) which has to be controlled by the control unit in connection with the nozzle, pintle and actuator.

Due to their compact design, pintle nozzles are the preferred solution. Independent on the design of the nozzle and valve, the materials of both have to withstand the extreme heat loads generated by the hot gas at high pressure. The pintle nozzles can be designed to have always a constant total nozzle throat area \( \sum A_{\text{throat}}(t) = \text{const.} \) This allows to connect the control of oppositely oriented nozzles directly in that way that the sum of both \( A_{\text{throat}}(t) \) is nearly constant, independent from the displacement of the pintle. By this, one actuator can drive the pintles for these two oppositely oriented nozzles.

### III. Functional Behavior of the Current ACS

The current ACS is meant to be applied within an interceptor against incoming ballistic missiles. Hence, it should be able to guarantee, that the interceptor is capable of hitting the target within a certain “miss distance” depending on the intercept velocity and misalignment of the interceptor with respect to the target.

In addition, the ACS should also be capable of stabilizing the missile attitude such, that a seeker is able to track the ballistic missile’s trajectory in order to enable the GNC to guide the missile properly to its predicted intercept point (PIP).

For this purpose, the current ACS uses four thrusters that have to provide the required thrust very accurately within a required thrust level and time frame. Depending on the missile and mission design, the ACS has to perform over a wide range of altitudes. In case the ACS is located outside the CoG, the missile maneuvering capability can be increased and the time constant for “instant” turns can be reduced.

The benefits of the current system are then
- compact system,
- attitude control in low and high altitudes
- hit to kill precision and
- a continuous operation of the system.

The concept is in its essence a thrust system with four nozzles in a quasi-Cartesian configuration.

The thrust direction is an integral element effectively controlled by an actuator system. Hot gas pintles are integrated to provide the control authority as part of the ACS. The actuators are directly coupled with the hot gas pintles which control the thrust direction of the ACS. In all cases, the actuators and their control are an integral element of the system.

If a constant mass flow gas generator, for example a solid fuel gas generator, is used, the use of four individually controlled ACS nozzles allows for significant savings of propellant mass. This is because only those ACS nozzles that need to provide thrust have to be opened while all other nozzles can be kept closed - thus preventing the loss of waste gas.

If a variable mass flow gas generator - like a gelled propellant gas generator or a RJ gas generator - is used the gas generator mass flow can be controlled through a valve additionally to the four controlled ACS nozzles.
Assuming that the gas is isentropic and the GG temperature is constant, the simplified gas supply dynamics can be described by the following ordinary differential equation:

\[
\frac{V_{\text{combustion}}(\cdot)}{RT_{\text{combustion}}} \frac{dP_{\text{combustion}}(\cdot)}{dt} = \frac{\rho_{\text{propellant}} A_{\text{burning}} T_{\text{burn}}}{RT_{\text{combustion}}} + \frac{\rho_{\text{propellant}} P_{\text{combustion}}(\cdot)}{RT_{\text{combustion}}} - \frac{P_{\text{combustion}}(\cdot) A_{\text{throttle}}}{c^*}. 
\]

The variables in the equation are explained in the nomenclature section at the end of this paper. The detailed dynamic description of the model is given in the mathematical model subsection (see (2)).

In order to show the capability of the system, the second component of the 3-dimensional Lorenz system has been chosen as a reference thrust tracking curve. The following simulation results use the Lorenz system

\[
\begin{align*}
\dot{\xi}_1(t) &= \xi_2(t) - \xi_1(t), &\xi_1(t) &= 1 \\
\dot{\xi}_2(t) &= \frac{28}{10} \xi_1(t) - \frac{1}{10} \xi_2(t) - \xi_1(t) \xi_3(t), &\xi_2(t) &= 0 \\
\dot{\xi}_3(t) &= \xi_1(t) \xi_2(t) - \frac{8}{30} \xi_3(t), &\xi_3(t) &= 3
\end{align*}
\]

The Lorenz system has the advantage that the system is fully deterministic, chaotic and bounded with bounded derivatives (see [SPAR1982], App. C), i.e. an estimation of the reference signal behavior in the future is not possible.

The simulation of the thrust performance of a coupled two-nozzle system assumes an actuator with a time constant of 40ms (for so called “bang bang” control) and a non-throttleable high-energetic solid propellant gas generator.

The graphs of the combustion chamber pressure, thrust and pintle position are normalized. Figure 3 shows the reference thrust profile (blue) for tracking, the simulated realized thrust (red) and the non-monotonic performance funnel (in-time calculated during simulation based on the tracked reference signal). An error feedback with an allowed maximum user defined tracking error of 5% is allowed.

Figure 4 presents the thrust error between commanded and simulated thrust and the performance funnel (first subplot). The second subplot contains the pintle movement. It is obvious that the performance funnel is non monotone and increases the performance funnel automatically to more than the pre-defined tracking error of 5% (see first subplot of Figure 4). This is around the zero thrust command (since the funnel boundary definition must be positive over the whole time). This is visible in Figure 3 with a zoom-in, too.

Figure 6 presents the combustion chamber pressure of the simulation and the simulated thrust – namely the generated thrust of nozzle 1 (blue), generated thrust of nozzle 2 (red) and the combined thrust (green) of both nozzles for the overall system (e.g. an ACS). Figure 5 shows the control input and the gain.

### IV. CONTROL ALGORITHMS

The design of the control algorithm is closely connected to the non-linear behavior of the system and its various operational phases, which require different control philosophies all realized within a single controller. Hence, the proper physical understanding of these phases and their accurate modeling can be regarded as the corner stone of the control algorithm.
Figure 4: simulation: thrust error and pintle position (normalized)

Figure 6: simulation: combustion pressure and thrust (normalized)

Figure 5: simulation: control input u and gain k
The main objective of the controller is to modify the controllable parameters (i.e. the actuator position) such that a certain command thrust is achieved without endangering the system’s physical stability.

The controller has to monitor and react to changes in the system even if the demand thrust remains unchanged. Typical examples are changes in material geometry and elastic deformations with implications on the thrust performance. This means that the control algorithms have to monitor constantly all the relevant system parameters and adapt the actuator position continuously.

The aim is to track a reference signal (e.g. the pre-defined thrust profile) in the context of input and output systems in the presence of input control. The investigated method is a pre-specified parameterized performance funnel. The tracking error is required to evolve within the funnel: transient and asymptotic behavior of the tracking error is influenced through the choice of parameter values which define the funnel. The proposed structure is an error feedback wherein the gain function evolves so as to preclude contact with the funnel boundary.

A feasibility condition (formulated in terms of the funnel data and the reference signal) can be formulated under which the tracking objective is achieved, whilst maintaining boundedness of all signals. The performance funnel evaluates system reliability and technical performance such that it allows a better orientation, more accurate matching and a better control of the parameter influence on the results.

[ESAIM2002] assumes arbitrary small time steps inside the proof of the main theorem. In real applications, the time step is limited to a minimum given by the sensor measurement system, actuator response time and the overall software/hardware delay.

Thus, a direct application of the funnel controller from [UVIL2010] is not possible. The well-known performance funnel concept was extended for use in applications with discrete time steps given by the used hardware.

V. MATHEMATICAL MODEL

Simulation models predict general system behavior as represented by speed, performance or reaction times by solving the respective equations of the system’s dynamics. The aim is to improve the operational effectiveness of a system through modeling and simulation. Figure 7 shows the model structure of the current ASC design which covers system dynamics, sensors to track the realized system thrust, a guidance law to generate the a-priori actuator commands, a propulsion model and a controller to achieve the desired thrust response to the guidance commands (actuator, pintle, thrust control).

As seen in Figure 7, the model is divided into different components – the propulsion gas supply, actuators, sensors, disturbances, thrust control, actuator controller and transformations. The closed-loop nature of the simulation shows the connectivity between the different model components. The propulsion supply system includes a propellant burning model, different flow models, description of the nozzle, pintle and actuator behavior as a function of time constants, combustion pressure, used propellant gas and ambient conditions. The system contains a large number of sub-components of which Figure 7 gives only a limited overview. The reference thrust follows a pre-specified profile, e.g. the chaotic 3-dimensional Lorenz system.

The kinematic and dynamic equations model the system under the influence of the actuator behavior, disturbances and the thrust forces produced by the propulsion supply system. Actuator commands and combustion pressure interaction and any other effects caused by actuators or signal processing are included.

In view of precision, speed and ability to respond of the ACS, the characteristics of the ACS and propulsion system are analyzed and computational tools are developed. Numerical simulations with a dynamic model are carried out to investigate the behavior.

In order to predict the thrust response of a controlled ACS, a mathematical model with several subclasses describes the dynamic of all components. The code can be easily integrated in applications or used as standalone program, e.g. when performing adapted performance simulations within a pre-existing database of actuator and nozzle data.

The dynamic equations of the model describe the performance and reaction time of the system (see (2)). The variables in (2) are explained in the nomenclature section at the end of this paper. Note that \( p_{\text{combustion}} \) and \( \dot{r}_{\text{burn}} \) depend on \( p_{\text{combustion}} \). For simplicity, since the correct mathematical space definition and initial values formulation are complex, the functional definition space of the right hand side of (2) is sloppy at this point.

Note that the equations of (2) are a non-linear state space representation of the thrust simulation model. The full non-linear equations are used, integrated and solved for pressure, mass flow, density and volume. In view of (2), some caution is required in formulating the existence of a unique global solution, i.e. defined on \( \mathbb{R}^{n+1} \). A straightforward formulation proves the existence and uniqueness of a global solution which is a major step for a prediction model. Standard theory of ordinary differential equations (see [WALT1998], Th. III.10.16) yields existence and uniqueness of a maximal solution of the initial value problem (2).
Traditional control techniques use a linearized state space form of (2) which may not be carried out in appropriate detail and may leave useful information/effects unconsidered.

The controller of the ACS thrust simulation model converts guidance commands into respective pintle position commands – more precisely, into actuator position commands to control the fuel mass flow direction from the gas supply system to the nozzles – in order to realize the required thrust level and direction needed for the intended missile maneuver. The resultant dynamics are measured by rate gyros, accelerometers and pressure sensors to design a non-adaptive feedback control system that tracks the a-priori defined thrust curve (e.g. the output of the chaotic Lorenz system).

As control system, the performance funnel is used [RTO2012]. The performance funnel ensures that the pre-specified transient behavior of the thrust generation and direction with respect to the reference profile is simpler than an adaptive controller insofar as the gain is not dynamically generated and does not invoke any internal model (see [RTO2012]).

VI. PERFORMANCE FUNNEL

Within this chapter the extension of the introduced funnel controller is discussed (see [IFAC2013], [RTO2012]) focusing on the practical, technical implementation of this control algorithm on an ACS in an uncontrolled environment.

A significant number of control algorithms that apply to dynamic inversion, backstepping techniques on simplified state variable models of the dynamics or PID controllers on linearized models in the frequency domain are presented in the literature (see [IEEE2004], [IEEE2007]). Here it is shown that if the practical non-linear time domain implementation is done correctly, even the simple non-monotone gain funnel controller can ensure the stability of the thrust direction and the thrust control (i.e. motor thrust, thrust direction and tracking error).

In the abstract sense, the complete model (2) can be described by functional differential equations in the presence of multi-input, multi-output systems which can be considered as a system of two interconnected nonlinear subsystems

\[
\begin{align*}
\dot{m}_{\text{combustion}}(\cdot) &= m_{\text{throat}}(\cdot) + \frac{d\rho_{\text{combustion}}(\nu_{\text{combustion}}(\cdot))}{dt} \\
\frac{dV_{\text{combustion}}(\cdot)}{dt} &= A_{\text{burning}}(\cdot)\nu_{\text{burn}}(\cdot) \\
\frac{dP_{\text{combustion}}(\cdot)}{dt} &= \left(\frac{c^*(\cdot)\nu_{\text{combustion}}(\cdot)^{2}}{V_{\text{combustion}}(\cdot)}\right)^2 \\
&= \left(\frac{c^*(\cdot)\nu_{\text{combustion}}(\cdot)^{2}}{V_{\text{combustion}}(\cdot)}\right)^2 \left(\rho_{\text{propellant}}(\cdot) - \rho_{\text{combustion}}(\cdot)\right)A_{\text{burning}}(\cdot)\nu_{\text{burn}}(\cdot)
\end{align*}
\]  

(2)


\[
\begin{align*}
\dot{x}(t) &= f_1(x(t), d_1(t)) + g(u(t)) , x(0) = x^0 \\
y(t) &= f_2(x(t), d_2(t))
\end{align*}
\]  

(3)

where \( f_1 : \mathbb{R}^n \times \mathbb{R}^p \rightarrow \mathbb{R}^n \), \( f_2 : \mathbb{R}^n \times \mathbb{R}^p \rightarrow \mathbb{R}^m \) are locally Lipschitz, \( d_1(\cdot) \in \mathcal{L}^m(\mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^p) \), \( d_2(\cdot) \in \mathcal{L}^m(\mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^m) \) are disturbances and \( g : \mathbb{R}^q \rightarrow \mathbb{R}^n \) is the continuous input function. In view of (3), some care is required in formulating the existence of a unique global solution, i.e. defined on \( \mathbb{R}_{\geq 0} \) which was discussed before. Momentarily regarding the second subsystem in (3), as an example for the presented application, \( y(\cdot) = F_{\text{thr}}(\cdot) \) is a possible representation.

The performance method is formulated in terms of the performance funnel (see Figure 8) \( F(\varphi) := \{(t, \eta) \in \mathbb{R}_{\geq 0} \times \mathbb{R}^m | \varphi(t) - \eta \| < 1 \} \), determined by a function \( \varphi(\cdot) \) which satisfies a Lipschitz condition. It has to be noted that the funnel boundary is given by \( \varphi(\cdot)^{-1} \) which contains a potential singularity. Typical funnel prototypes are illustrated in Figure 8.

![Figure 8: Prototypes of non-/symmetric performance funnels](image)

The control objective is as follows. Determine a feedback structure which ensures that, for a given reference signal \( y_{\text{ref}}(\cdot) \in W^{1,\infty}(\mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^m) \), the output tracking error \( e(\cdot) := y(\cdot) - y_{\text{ref}}(\cdot) \) evolves within the funnel, i.e. \( \{(t, e(t)) \}_{t \in \mathbb{R}_{\geq 0}} \subseteq F(\varphi) \): transient and asymptotic behavior of the tracking error is influenced through choice of the function \( \varphi(\cdot) \). The proposed structure is an error feedback wherein the gain function evolves so as to preclude contact with the funnel boundary.
The performance funnel was first introduced in [ESAIM2002] and [UVIL2010] with the goal of achieving better performances than PID controllers in control situations. This is achieved with a basically simpler representation, as unlike a PID controller the funnel does not require a differential equation or internal module. An approach to the simulation model of the performance estimation for the purpose of formal verification and analysis is illustrated.

The performance funnel ensures pre-specified transient behavior of the tracking error, has a non-monotone gain, is simpler than an adaptive controller (insofar as the gain is not dynamically generated), and does not invoke any internal model. The method combines the benefits of two different approaches: the simplicity of adaptive feedback control and an intrinsic high-gain property if necessary. It can be applied directly to the problem without differential equations or an internal model. Two performance evaluation methods are implemented – Euclidean and component wise analysis. The first method uses the Euclidean norm of the output signal (see Figure 8). The second part fits each output signal separately. An allowed corridor for the input is used to fit the reference signals. The simple implementation of the algorithm only uses the input and reference signals.

A wide variety of funnels are possible (see Figure 8). Monotonicity of the funnel boundary is not required. Non-monotone funnels may be advantageous in situations for which it is known a priori when perturbations or set-point changes may occur – in this sense; non-monotone funnels have the connotation of re-initialization of the control structure.

The simplified workflow of the funnel based performance approach for an input output system is shown schematically in Figure 9.

VII. TEST FACILITY DEMONSTRATOR AND TRIALS

Based on requirements like performance, operational time, installation space, total mass etc., a first principal operational ACS design was developed to fulfill the requirements (see Figure 1 and Figure 2). The preliminary concept uses in-house tools, CFD calculations and experimental results for a principle concept of a gas supply system with an actuator concept to move the pintle and meet the performance requirements.

This basic design with all components was implemented and integrated into a simulation environment in Matlab/Simulink including propellant burning model, empirical actuator behavior curves, nozzle and pintle shapes to realize a simulation and performance prediction model to simulate the overall performance of the system, to identify and optimize key driver components, to do sensitivity studies (e.g. influence of actuator time constant or influence of sensor measurement and data transfer rates etc.) and to validate and verify test facility trials and results.

Figure 9: Schematically flow of the funnel based thrust direction control

Figure 10: ACS - test facility hardware design

Based on this lightweight design, a test facility hardware concept design (see Figure 10) was generated to fulfill the function of an ACS, but it is more modular and less integrated than a possible flight design to allow the change or modification of parts and has more massive structures in order to allow refurbishment of parts or assemblies or re-use.
The demonstrator concept is represented by two nozzles for the ACS, i.e. one nozzle plane, with plenum, pintles and an actuator. The full functionality of the ACS can be demonstrated with only 50 per cent of the nozzle numbers and actuators.

The nozzles have a quasi-Cartesian configuration with pintles to control the thrust direction. The actuators are not exposed to the hot gas. Typical test facility actuators are used, i.e. the time constant of the demonstrator is slower than expected for an operational system.

The aims of the test facility trials are to determine the dynamic behavior of the system, validate the accuracy of the design and simulation model, determine actuator forces and confirm the design and technology handling.

For simplicity, coupled pintles are used instead of individual controlled nozzles. This implies a higher propellant mass flow during the trial but reduces the complexity of the system. A complete closure of a nozzle is a harder challenge then a “nearly closed”. Therefore, “nearly closed” nozzles are assumed.

The simulation model and funnel controller implementation is verified by analytical test cases and real test facility trials with an ACS test facility demonstrator based on a flight design. In order to investigate the performance and control algorithm of the controlled ACS, a single trial will be chosen.
For a standardized classical missile, proportional navigation is a first standard approach for a guidance law and performance studies (e.g. like range, flight time, footprint etc.). The presented trial uses a common thrust reference profile as expected during real flights (see Figure 11). Since the ignition process is not represented inside the simulation model, the reference thrust has the zero command at the beginning to guarantee that the ignition process of the engine is finished.

As a gas supply system, a gelled propellant with a combustion temperature of approximately 2000 K is used since the propellant handling is simpler and cheaper compared to a solid propellant gas generator. For safety reasons, if a failure happens during the trial, the gelled propellant combustion and gas production can be stopped immediately.

The graphs of the combustion chamber pressure, thrust and pintle position are normalized. Figure 12 shows the simulated combustion chamber pressure (blue) based on the simulation model and the measured trial pressure (magenta). The ignition process is not represented inside the model which explains the differences between measurement and simulation over the first 0.4s. Afterwards, the fitting between measurement and simulation is good and the pressure movements are visible in both graphs. Some loss of insulation explains the pressure drop at the end of the trial.

Figure 14 shows the commanded, pre-defined reference thrust (blue) as in Figure 11 which shall be tracked by the ACS. The measured thrust of the active controlled system via pintle position and thrust control is in magenta taking into account that the used controller is an inline pre-defined
performance funnel. As seen in Figure 8, the test facility demonstrator has an oscillation capability and no integrated damper which would have an influence to the thrust measurement. Therefore, the main focus is not on the measured thrust curve in absolute values but on the qualitative behavior and the control handling and reaction. It can be seen that the introduced funnel controller can guarantee the system control, material dispersions (e.g. insulation loss, erosion etc.), oscillation events and actuator delays.

Figure 14 presents the thrust error between commanded and measured thrust and the performance funnel (first subplot). Through the oscillation capability and no integrated damper, the user allowed pre-defined error range is 8%. Most of the time, the tracking error is less than 4%. As explained in subsection III, the performance funnel is non monotone and the algorithm increases the performance funnel automatically to more than the pre-defined tracking error (see first subplot of Figure 14).

The second subplot contains the pintle movement. Note that a mechanical actuator has a reaction time for a new commanded position.

Figure 15 shows the control input and the gain. A zoom-in shows that the control input \( u(t) \) is smaller than 0.05 in absolute values. This means that the tracking is fast and immediately such that no large control inputs are necessary. Moreover, the gain is not constant and the intrinsic high gain property is obviously, i.e. the gain is large if necessary and otherwise small.

VIII. CONCLUSION

The combination of increasingly complex and expensive hardware and constrained military budgets results in the fact that it is not economically feasible to test a system for the entire possible range of operations or with different designs. A precise model of the performance, certified by trials that can be used to predict system performance accurately for all operating points or updated subcomponents is essential. Especially the controller design and the modeling accuracy and reliability are very important constituents of the missile or rocket engineering design.

This paper presented the main components of a simulation control system applied to an ACS. A main issue was to prove that such a complex system can be controlled using a pre-defined performance funnel. A linearized simulation model, transfer functions or frequency-domain descriptions are not necessary.

The tracking error is required to evolve within the funnel: transient and asymptotic behavior of the tracking error is influenced through choice of parameter values which define the funnel. The proposed structure is an error feedback wherein the gain function evolves so as to preclude contact with the funnel boundary. A feasibility condition (formulated in terms of the funnel data and the reference signal) can be formulated under which the tracking objective is achieved, whilst maintaining boundedness of all signals. The performance funnel evaluates system reliability and technical performance so as to allow a better orientation, more accurate matching and a better control of the parameter influence on the results.

The presented performance funnel controller guarantees a transient behavior for the missile. Since the performance method is described by generic functions, a large range of models can be analyzed. The implemented funnel algorithm allows the optimization of both – the geometrical shape of the pintle nozzle system and actuator parameters with respect to given requirements.

As explained in the algorithmic description, the maximum allowed error of the tracked parameter (e.g. thrust) is user defined. The maximum allowed error has to be high enough as that a physically meaningful solution can be obtained.
NOMENCLATURE

ACs  Attitude Control System
CoG  Centre of Gravity
GG   Gas Generator
PID  Proportional Integral Differential (Controller)
PN   Proportional Navigation
RJ   Ram Jet
TDR  Throttleable Ducted Rocket

$A_{\text{burning}}(\cdot)$ burning area of the propellant
$A_{\text{throat}}(\cdot)$ pintle position dependent nozzle throat area
$c^*(\cdot)$ characteristic velocity of the gas
$F_{\text{thr}}(\cdot)$ thrust
$\gamma$ heat specific value
$\dot{m}_{\text{combustion}}(\cdot)$ mass flow rate due to propellant combustion
$\dot{m}_{\text{throat}}(\cdot)$ mass flow rate exiting the nozzle
$r_{\text{burn}}(\cdot)$ burn rate of the propellant
$P_{\text{combustion}}(\cdot)$ combustion chamber pressure
$R$ universal gas constant
$\rho_{\text{propellant}}$ density of the propellant
$\rho_{\text{combustion}}$ gas density in the combustion chamber
$T_{\text{combustion}}(\cdot)$ combustion chamber temperature
$V_{\text{combustion}}(\cdot)$ combustion chamber volume
$\mathbb{R}, \mathbb{R}_{\geq 0}$ space of (non-negative) real numbers
$\mathcal{L}^\infty(\mathbb{R}_{\geq 0} \to \mathbb{R})$ measurable and essentially bounded functions
$W^1(\mathbb{R}_{\geq 0} \to \mathbb{R})$ bounded locally absolutely continuous functions with essentially bounded first derivative

REFERENCES


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