Journal of Energy Challenges and Mechanics
http://www.nscj.co.uk/JECM/
ISSN 2056-9386
Volume 3 (2016) issue 1, article 1

On the comparison among optimal measurement placement methods for a hybrid micro grid harmonic state estimation. Part I: theoretical aspects

关于混合微电网谐波状态估计的最佳测量选择方法之比较。第一部分：理论方面

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Accepted for publication on 12th October 2015

Abstract - **The dynamic state estimation is an important issue in the management and control of micro grids (μGs), since given the existence of distributed energy resources and time-varying, non-linear loads. In this paper, dynamic harmonic state estimation (DHSE) is used to evaluate the distortions of the voltage and current waveforms caused by the presence of time-varying, non-linear loads in hybrid AC/DC μGs. A hybrid μG includes controllable and non-controllable loads, linear and non-linear loads, dispatchable and non-dispatchable dispersed generation units (such as a photovoltaic system and a gas micro-turbine) and energy storage systems. DHSE requires that voltage and current measurements be taken in correspondence with particular buses and lines of the system, which must be selected appropriately to guarantee that the system is observable. In this paper, we selected two techniques from the relevant literature for the optimal placement of the measurement units, and they were used for the DHSE with a Kalman filter (KF) on the hybrid AC/DC μG. The first method was based on the application of integer linear programming, and the second method was based on the minimum condition number of the measurement matrix. This paper reports the theoretical aspects of the methods that were used, and it is a companion paper to the Part II paper in which the results of the numerical experiments are presented.**

Keywords - **Optimal measurement placement, dynamic harmonic state estimation, Kalman filter, micro grid, power quality**

I. INTRODUCTION

To date, the structure and management of power distribution systems have been modified and improved significantly due to the development of the new concepts of smart grids (SGs) and micro grids (μGs). The increasing level of penetration of distributed generation, storage systems, and controllable loads is the main cause of these changes; the management of these new systems is guaranteed by the use of information and communication technologies [1-5]. In particular, μGs can be classified as either AC or DC μGs based on the supply voltage [2-4]. AC μGs utilize the existing AC grid technologies, but DC μGs can be used to connect the distributed generation sources that generate DC power. To gain the advantages of both types of grids, hybrid AC/DC μGs have been proposed and developed recently, especially in industrial contexts in which dispatchable and renewable generation units, storage systems and controllable loads actively contribute to the operation of the electrical system [5]. The hybrid AC/DC μGs are the main focus of this paper.

The optimal operation of hybrid μGs requires careful consideration of both the technical and economic aspects of such systems, i.e., they must simultaneously satisfy increasing needs in terms of required power quality (PQ) and minimize costs. Optimal management usually is achieved by using a centralized control system (CCS) that operates based on estimates of the state of the system.

The estimates of the state of the system are used as input data for different tasks; for example, in [6], the estimates were used to compensate accurately for the harmonic disturbances, while, in [7], they were used for the real-time control of active and reactive power. In this paper, we dealt with the dynamic harmonic state estimation (DHSE) with the aim of estimating

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the time-varying distortions of waveforms in the hybrid μG due to the presence of non-linear loads.

DHSE has been investigated extensively in the relevant literature, but the investigations have been focused mainly on AC networks [6, 8-14].

Specifically, in [8], a comprehensive review of the existing techniques was presented, specially focusing on neural network methods, which require long processing times. In [9, 10] the weighted least-squares estimator was used to minimize the error between the estimates and the measurements of the variables.

The Kalman filter (KF) was used in [6, 11, 12]. However, since the number of available measurements is usually smaller than the number of measurements required to determine the harmonic state of the network completely, the equation system that must be solved is usually underdetermined. Thus, it is essential to evaluate the observability of the system [13, 14].

It is well known that DHSE usually requires input data that consist of voltage and current measurements taken from opportunely selected buses and lines of the grid. The measurement units (MUs) fit for the purpose must fulfil specific requirements, especially in terms of sampling rates, synchronization of the measurements, and accuracy of the current and voltage transducers [15]. Phasor measurement units (PMUs), which have been studied extensively for high-voltage, AC transmission networks, seem to be among the most suitable devices for satisfying the aforesaid requisites. Their use in distribution networks is expected to increase significantly in the next few years due to the reductions of the manufacturing and installation costs that have resulted from technological improvements and the economies of scale [15, 16].

In the relevant literature, many techniques for the optimal allocation of MUs have been proposed for AC networks [17-26]. These techniques also can be used to collect the measurements required for the DHSE [27].

In [17-20], these techniques were classified on the basis of the algorithms that were used. The methods based on genetic algorithms are, substantially, adaptive heuristic research algorithms, that emulate natural evolution processes [21, 22]. These methods are very adaptive and robust, but they require long processing times; thus, they cannot be used when there are needs to reallocate MUs, which can arise when inevitable failures occur in obtaining measurements. In [23], particle swarm optimization was proposed for the placement of MUs. This method achieves the optimal placement of MUs, but it has the disadvantages of not accounting for the computational burden, not considering contingencies, and the lack of ease and versatility of implementation. In [24], an iterative procedure based on a binary research algorithm was proposed. This algorithm performs an exhaustive search by considering the possibility of faults in each line, but it is characterized by a heavy computational burden. In [25], a method was proposed based on the criteria of the minimum condition number of the measurement matrix. In [26], a fast, versatile technique was proposed based on integer linear programming (ILP); it solves a binary, linear programming problem to guarantee the observability of the system.

In this paper, we propose to achieve DHSE on a hybrid AC/DC μG through a KF-based approach [6, 11, 28]. In this approach, time domain values of current and voltage measurements are required as inputs, and they are supplied by adequate MUs placed on the basis of the techniques proposed in [25, 26].

In the companion paper [29], numerical applications were performed on an AC/DC μG proposed for an actual industrial facility in southern Italy, and a comparison of the proposed approaches was developed on the basis of: (i) the number of measurements required to guarantee the observability of the system; (ii) the accuracy of the corresponding KF-based DHSE; and (iii) the computational burden.

This paper is organized as follows. Section II describes the architecture and the components of the hybrid μG that was considered. In Section III, the proposed optimal methods for the placement of the MUs are presented, and in Section IV the KF-based approach for the DHSE is presented. Our conclusions are reported in Section V.

II. THE HYBRID AC/DC MICRO GRID UNDER STUDY

The general scheme of the hybrid AC/DC μG under study is shown in Fig. 1. The μG included linear and non-linear loads, dispatchable and non-dispatchable distributed generation units, and energy storage systems. A DC/AC static converter allowed the connection of the DC part of the μG (where non-dispatchable DC generators, sensitive loads, and energy storage systems are set) to the AC part of the μG (where dispatchable AC generators and linear/non-linear AC loads are set).

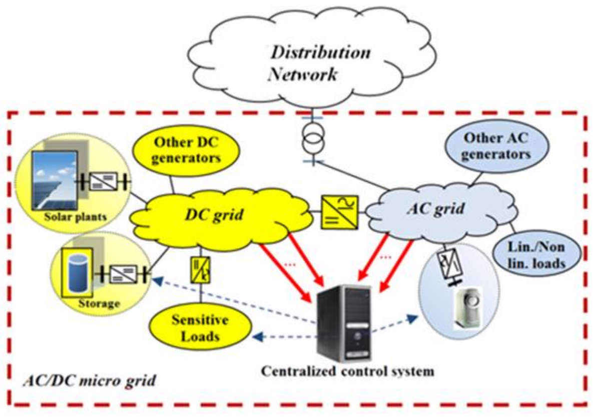

Fig. 1, The hybrid AC/DC μG general scheme.

Note that:

- sensitive AC loads are connected to the DC side of the μG through DC/AC static converters;
- DC generators are non-dispatchable photovoltaic systems that are connected to the DC grid through DC/DC static converters and equipped with a maximum power point tracker (MPPT) control system;

- the storage system is a battery that is connected to the DC grid through a DC/DC static converter;
- the AC generators, which are dispatchable generation units, are gas micro-turbines that are connected to the AC grid through AC/AC static converters.

The CCS in Fig. 1 performs the control strategy for the optimal operation of the entire system, and it operates on the basis of a dynamic estimation of the state of the system. The red arrows in Fig. 1 represent the measurements taken from the AC and DC sides of the µG. These measurements were selected on the basis of the optimal MUs placement techniques, reported in the following Section.

III. PROPOSED OPTIMAL MUS PLACEMENT METHODS

In this paper, we used the *minimum condition number method (MCNM)* and the *integer linear programming method (ILPM)* proposed for AC networks in [25] and [26], respectively, in the case of the hybrid µG shown in Fig. 1. For the grid that we were considering, these methods were considered to be among the best procedures in terms of flexibility and computing time. We assumed that each MU was a multi-channel device that could measure the bus voltage in the bus in which it was installed and measure all of the line currents linked to that bus. Also, we assumed the MUs could fulfil the aforesaid requirements in terms of sampling rates, synchronization of the measurements, and the accuracies of the current and voltage transducers.

3.1. MINIMUM CONDITION NUMBER METHOD

This method selects the minimum number of electrical variables that must be measured in order to guarantee the observability of the electrical system that is being assessed. It is based on the criterion of minimum condition number of the measurement matrix, which is obtained from the equation system that links the measurements to the unknown variables that must be estimated.

The condition number of a matrix is defined starting from the knowledge of the singular values of the matrix. In fact, considering a generic matrix $\boldsymbol{T}$ and indicating its conjugate transpose by $\boldsymbol{T}^*$, the singular values of $\boldsymbol{T}$ are the square roots of the eigenvalues of the matrix $\boldsymbol{T}^*\boldsymbol{T}$. The condition number of a matrix is the ratio of the maximum singular value to the minimum singular value of the matrix.

In the phasor domain, for each harmonic order h, let $\overline{\boldsymbol{Z}}(h)$ be the measurements vector, $\overline{\boldsymbol{X}}(h)$ be the unknown vector of state variables to be estimated, and $\overline{\boldsymbol{E}}(h)$ be the measurement errors vector; it is possible to define the *measurement matrix* $\dot{\boldsymbol{H}}(h)$ from the mathematical model that links $\overline{\boldsymbol{Z}}(h)$ and $\overline{\boldsymbol{X}}(h)$ as follows:

$$\overline{\boldsymbol{Z}}(h) = \dot{\boldsymbol{H}}(h)\overline{\boldsymbol{X}}(h) + \overline{\boldsymbol{E}}(h) \tag{1}$$

Once $\dot{\boldsymbol{H}}(h)$ is determined, first an iterative procedure can be applied to determine the variables that must be measured, and, then, the number of MUs to be placed can be minimized through an exhaustive procedure.

It is well known that the measurement matrix $\dot{\boldsymbol{H}}(h)$ can be obtained from the relationships that link the bus voltages (which are the unknown variables to be estimated) to the load currents, line currents, or some other bus voltages (which can be measured).

If we let N be the number of buses, the relationship between the $[N\text{x}1]$ complex vector $\overline{\boldsymbol{I}}_{\boldsymbol{N}}(h)$ of load currents at each bus and the $[N\text{x}1]$ complex vector $\overline{\boldsymbol{V}}_{\boldsymbol{N}}(h)$ of bus voltages is given by:

$$\overline{\boldsymbol{I}}_{\boldsymbol{N}}(h) = \dot{\boldsymbol{Y}}_{\boldsymbol{NN}}(h)\overline{\boldsymbol{V}}_{\boldsymbol{N}}(h) \tag{2}$$

where $\dot{\boldsymbol{Y}}_{\boldsymbol{NN}}(h)$ is the $[N\text{x}N]$ admittance matrix.

Obviously, the relationship between the complex vector $\overline{\boldsymbol{V}}_{\boldsymbol{N}}(h)$ of bus voltages and itself is given by:

$$\overline{\boldsymbol{V}}_{\boldsymbol{N}}(h) = \boldsymbol{I}_{\boldsymbol{NN}}\overline{\boldsymbol{V}}_{\boldsymbol{N}}(h) \tag{3}$$

where $\boldsymbol{I}_{\boldsymbol{NN}}$ is the $[N\text{x}N]$ identical matrix.

The relationship between the $[L\text{x}1]$ complex vector $\overline{\boldsymbol{I}}_{\boldsymbol{L}}(h)$ of the line currents and the complex vector $\overline{\boldsymbol{V}}_{\boldsymbol{N}}(h)$ of the bus voltages is given by:

$$\overline{\boldsymbol{I}}_{\boldsymbol{L}}(h) = \dot{\boldsymbol{Y}}_{\boldsymbol{LN}}(h)\overline{\boldsymbol{V}}_{\boldsymbol{N}}(h) \tag{4}$$

where $\dot{\boldsymbol{Y}}_{\boldsymbol{LN}}(h)$ is the $[L\text{x}N]$ line-bus admittance matrix.

In the most general case in which all load currents, all line currents, and all bus voltages can be measured, the measurement matrix $\dot{\boldsymbol{H}}(h)$ in eq. (1) is given by the combination of the matrices $\dot{\boldsymbol{Y}}_{\boldsymbol{NN}}(h)$, $\boldsymbol{I}_{\boldsymbol{NN}}$, and $\dot{\boldsymbol{Y}}_{\boldsymbol{LN}}(h)$ in eqs. (2), (3), and (4), respectively; the size of matrix $\dot{\boldsymbol{H}}(h)$ is $[M\text{x}N]$, where $M = 2N + L$ is the maximum possible number of measurements.

An iterative procedure can be used to determine the minimum number of variables to be measured. Starting from the full-measurement matrix, at the first iteration, each possible measurement is excluded by eliminating the corresponding row in the matrix, $\dot{\boldsymbol{H}}(h)$; then, the different condition number is calculated for each reduced matrix, i.e., $\dot{\boldsymbol{H}}_{\boldsymbol{1}}(h), \dot{\boldsymbol{H}}_{\boldsymbol{2}}(h), \dots, \dot{\boldsymbol{H}}_{\boldsymbol{M}}(h)$, obtained by respectively deleting row number $1,2,\dots,M$ from $\dot{\boldsymbol{H}}(h)$. The matrix that presents the lower condition number (i.e., $\dot{\boldsymbol{H}}_{\boldsymbol{j}}(h)$) is chosen as the subject of the next iteration; each possible measurement is excluded by eliminating the corresponding row in the matrix $\dot{\boldsymbol{H}}_{\boldsymbol{j}}(h)$, the different condition number for each reduced matrix $\dot{\boldsymbol{H}}_{\boldsymbol{j_1}}(h), \dot{\boldsymbol{H}}_{\boldsymbol{j_2}}(h), \dots, \dot{\boldsymbol{H}}_{\boldsymbol{j_{M-1}}}(h)$ is calculated, and the matrix that has the lowest condition number is chosen again as the subject of the next iteration. The whole iterative process is stopped when the size of the selected matrix is $[N\text{x}N]$.

After the variables that are to be measured have been determined, we must determine the minimum number of MUs that must be installed in order to obtain those measurements in the most economical way possible. In order to minimize the number of MUs, the best choice is to employ multi-channel devices and to select their positions by a heuristic procedure, as described below:

1. when the measurement of a bus voltage or a load current is determined to be necessary in the above depicted iterative procedure, the corresponding bus is called the "major bus;" all other buses are called "minor buses," and a MU is placed in all of the major buses;
2. if it is necessary to measure at least one of the line currents connected to a major bus, a channel of the MU placed in that major bus must be dedicated to measure that line current;
3. then, a MU is placed in the minor bus connected to the highest number of lines in which currents must be measured; this bus no longer can be considered to be suitable for the placement of a MU;
4. step 3 is repeated until none of the remaining minor buses is linked to a line in which current must be measured and is not yet measured by another MU.

In [25], the aforesaid approach was applied for each harmonic order of interest. In this paper, since the considered MUs acquire waveform samples in the time domain, the iteration of the aforesaid procedure at each harmonic order appeared to be redundant. Therefore, supported by the experimental applications reported in the companion paper [29], we propose to apply the procedure to the measurement matrix evaluated only for one appropriately-selected harmonic order.

3.2. Integer Linear Programming method

The method based on Integer Linear Programming is a fast, versatile technique based on the solution algorithm of a programming problem in which an objective function of integer variables is minimized under linear constraints. In [26], this method was proposed for the state estimation at the fundamental component, while, in this paper, it was used to get reliable state estimation for all of the harmonics considered in the hybrid μG shown in Fig.1.

The entire system is observable by allocating the right number of MUs in strategic buses. If there are N buses in the AC side of the μG, the problem of the optimal allocation of MUs can be solved by the formulation of an optimization problem in integer variables. The objective function to be minimized is:

$$f_{obj} = \sum_i^N w_i \cdot b_i \tag{5}$$

and the inequality constraints to be satisfied are:

$$\boldsymbol{f}(\boldsymbol{b}) \geq \widehat{\mathbf{1}} \tag{6}$$

where: $\boldsymbol{b} = \{b_1, b_2, \dots, b_N\}$ is a vector of decisional binary variables, with $b_i = 1$ if a MU is placed in the i-th bus of the grid, otherwise $b_i = 0$; w_i is a weight factor, which can be related to the cost of the single MU installed at the i-th bus; $\widehat{\mathbf{1}}$ is the unity vector; $\boldsymbol{f}(\boldsymbol{b})$ is a vector function that is linked to the connectivity matrix of the system obtained from the non-oriented graph of the electrical circuit of the grid. The i-th component of this function expresses the observability of the i-th bus voltage: it is different from zero if, based on the interpretation of the graph through logical operators, at least one MU is placed in the i-th bus and/or in its nearby, linked buses.

The elements, $S_{k,m}$, of the binary connectivity matrix $\boldsymbol{S}$ are defined as follows:

$$S_{k,m} = \begin{cases} 1 & if\ k = m \\ 1 & if\ the\ k^{th}\ and\ the\ m^{th}\ buses\ are\ linked \\ 0 & otherwise \end{cases} \tag{7}$$

The connectivity matrix $\boldsymbol{S}$ can be identified directly by inspection of the electrical circuit or through the analysis of the bus admittance matrix by transforming its values in binary values. The vector function $\boldsymbol{f}(\boldsymbol{b})$ is obtained as:

$$\boldsymbol{f}(\boldsymbol{b}) = \boldsymbol{S}\,\boldsymbol{b} \tag{8}$$

where $f_i = S_{i1}b_1 + \cdots + S_{iN}b_N$ is constrained to be not less than unity, i.e., $f_i \geq 1$. Note that the operator "+" is intended as the logical operator "OR", and the value "1" on the right of each inequality ensures that at least one of each variable that appears in the "sum" on the left is different from zero.

Note that the ILP method is based only on the inspection of the topology of the grid, so the results obtained from the optimal MUs placement are still valid even if these devices provide measurements in the time domain.

IV. Dynamic harmonic system state estimation

The DHSE procedure proposed in [6,11,28] for AC networks was applied to the case of the hybrid μG in Fig.1; this technique is based on the application of KF and considers the measurements provided by the MUs placed by applying both of the methods presented in Section III.

Since the only non-linear elements of a μG are the static converters, a linearized model of the system can be performed by modelling the non-linear load as harmonic current injection. In this way, the state-space model can be expressed through a matrix equation. Let $\boldsymbol{x}$ be the state vector, $\boldsymbol{u}$ be the controllable input vector, $\boldsymbol{d}$ be the non-controllable input vector, and $\boldsymbol{y}$ be the output vector. In particular, for the AC side of the μG, $\boldsymbol{x}$ consists of the inductor currents and capacitor voltages, $\boldsymbol{u}$ consists of the controllable current injections, $\boldsymbol{d}$ consists of the disturbance currents, and $\boldsymbol{y}$ consists of the busbar voltages. Then, the state-space model is:

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{F}\boldsymbol{d} \\ \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \end{cases} \tag{9}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{F}$, and $\boldsymbol{C}$ are matrices that can be obtained from the equivalent circuit of the μG.

In this paper, the disturbances $\boldsymbol{d}$ in eq. (9) were modelled by harmonic current injections, caused by the non-linear loads of the grid [12]. By this model, the h-th current harmonic injected in the n-th bus can be expressed as follows:

$$d_{n,h}(t) = D_{n,h}\cos\left(\omega_h t + \varphi_{n,h}\right) \tag{10}$$

In order to express a dynamic disturbance model coherently with eq. (9), the following form can be stated for the h-th current harmonic:

$$\dot{\boldsymbol{z}}_{n,h} = \begin{bmatrix} \dot{d}_{n,h} \\ \ddot{d}_{n,h} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_h^2 & 0 \end{bmatrix} \cdot \begin{bmatrix} d_{n,h} \\ \dot{d}_{n,h} \end{bmatrix} = \boldsymbol{A}_{z_h} \boldsymbol{z}_{n,h} \qquad (11)$$

where $\boldsymbol{z}_{n,h}$ is the vector that includes the disturbance $d_{n,h}$ in (10) and its first derivative $\dot{d}_{n,h}$.

Now, let $\boldsymbol{h} = [d_{1,1}, \dots, d_{N,1}, \dots, d_{1,H}, \dots, d_{N,H}]$ be the vector of the harmonic disturbances, H be the number of the considered harmonics, and $\boldsymbol{G}$ be the matrix defined as:

$$\boldsymbol{G} = \begin{bmatrix} -\omega_1^2 \boldsymbol{I_N} & \boldsymbol{0_N} & \cdots & \boldsymbol{0_N} \\ \boldsymbol{0_N} & -\omega_2^2 \boldsymbol{I_N} & \cdots & \boldsymbol{0_N} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0_N} & \boldsymbol{0_N} & \cdots & -\omega_H^2 \boldsymbol{I_N} \end{bmatrix} \qquad (12)$$

it results:

$$\dot{\boldsymbol{z}} = \begin{bmatrix} \dot{\boldsymbol{h}} \\ \ddot{\boldsymbol{h}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0_M} & \boldsymbol{I_M} \\ \boldsymbol{G} & \boldsymbol{0_M} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{h} \\ \dot{\boldsymbol{h}} \end{bmatrix} = \boldsymbol{A}_z \boldsymbol{z} \qquad (13)$$

where $\boldsymbol{0_N}$, $\boldsymbol{0_M}$, $\boldsymbol{I_N}$, and $\boldsymbol{I_M}$ are the null and identity square matrices of specified sizes, with $M = H \cdot N$.

Eqs. (9) and (13), disturbance component vector $\boldsymbol{h}$, and its derivative $\dot{\boldsymbol{h}}$, together with the state variables $\boldsymbol{x}$, form the expanded state variables $\boldsymbol{\xi}$ of the dynamic time-invariant system; therefore, it is:

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{z}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{F_z} \\ \boldsymbol{0} & \boldsymbol{A_z} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} + \begin{bmatrix} \boldsymbol{B} \\ \boldsymbol{0} \end{bmatrix} \cdot \boldsymbol{u} \qquad (14)$$

where $\boldsymbol{F_z}$ is the matrix that relates the derivative state vector $\dot{\boldsymbol{x}}$ to the vector $\boldsymbol{z}$, and it is constituted by H submatrices equal to the matrix $\boldsymbol{F}$ and H null submatrices $\boldsymbol{0_F}$ having the same size of $\boldsymbol{F}$, i.e., if $H = 3$, $\boldsymbol{F_z} = [\boldsymbol{F}\ \boldsymbol{F}\ \boldsymbol{F}\ \boldsymbol{0_F}\ \boldsymbol{0_F}\ \boldsymbol{0_F}]$. Eq. (14) can be expressed in a more compact form as:

$$\dot{\boldsymbol{\xi}} = \widetilde{\boldsymbol{A}} \cdot \boldsymbol{\xi} + \widetilde{\boldsymbol{B}} \cdot \boldsymbol{u} \qquad (15)$$

The relationship that links the expanded state variables $\boldsymbol{\xi}$ to measurement vector $\boldsymbol{\zeta}$ is:

$$\boldsymbol{\zeta} = \widetilde{\boldsymbol{C}} \cdot \boldsymbol{\xi} \qquad (16)$$

where $\widetilde{\boldsymbol{C}}$ is normally a sparse matrix in which the non-zero elements are unity and correspond to the variables of $\boldsymbol{\xi}$ that must be measured.

Now, let us assume that measurements are available for the dynamic state estimation at the generic time $t_k = t_0 + kT_s$, where T_s is the sample time; the model of the system described by eqs. (15) and (16) can be formulated in discrete-time form, as follows:

$$\begin{cases} \boldsymbol{\xi_k} = \widetilde{\boldsymbol{A}}_{\boldsymbol{d}} \cdot \boldsymbol{\xi_{k-1}} + \widetilde{\boldsymbol{B}}_{\boldsymbol{d}} \cdot \boldsymbol{u_{k-1}} + \boldsymbol{w_{k-1}} \\ \boldsymbol{\zeta_k} = \widetilde{\boldsymbol{C}} \cdot \boldsymbol{\xi_k} + \boldsymbol{v_{k-1}} \end{cases} \qquad (17)$$

where $\boldsymbol{\xi_k} = \boldsymbol{\xi}(t = t_k)$, $\widetilde{\boldsymbol{A}}_{\boldsymbol{d}} = exp(\widetilde{\boldsymbol{A}}\, T_s)$, and $\boldsymbol{w_k}, \boldsymbol{v_k}$ are process noise and measurement noise, usually assumed as independent, white noise, respectively, characterized by a Gaussian probability density function, and:

$$\widetilde{\boldsymbol{B}}_{\boldsymbol{d}} = \widetilde{\boldsymbol{A}}^{-1} \cdot \left[exp(\widetilde{\boldsymbol{A}}\, T_s)\right) - \boldsymbol{I}] \cdot \widetilde{\boldsymbol{B}} \qquad (18)$$

Starting from eq. (17), the recursive equations of KF can be expressed as:

- two *predictive time-update* equations:

$$\hat{\boldsymbol{\xi}}_{\boldsymbol{k|k-1}} = \widetilde{\boldsymbol{A}}_{\boldsymbol{d}} \cdot \hat{\boldsymbol{\xi}}_{\boldsymbol{k-1|k-1}} + \widetilde{\boldsymbol{B}}_{\boldsymbol{d}} \cdot \boldsymbol{u_{k-1}} \qquad (19)$$

$$\boldsymbol{P_{k|k-1}} = \widetilde{\boldsymbol{A}}_{\boldsymbol{d}} \cdot \boldsymbol{P_{k-1|k-1}} \cdot \widetilde{\boldsymbol{A}}_{\boldsymbol{d}}^{\boldsymbol{T}} + \boldsymbol{W} \qquad (20)$$

- three *corrective measurement-update* equations:

$$\boldsymbol{K_k} = \boldsymbol{P_{k|k-1}} \cdot \widetilde{\boldsymbol{C}}^{\boldsymbol{T}} \cdot (\widetilde{\boldsymbol{C}} \cdot \boldsymbol{P_{k|k-1}} \cdot \widetilde{\boldsymbol{C}}^{\boldsymbol{T}} + \boldsymbol{V})^{-1} \qquad (21)$$

$$\hat{\boldsymbol{\xi}}_{\boldsymbol{k|k}} = \hat{\boldsymbol{\xi}}_{\boldsymbol{k|k-1}} + \boldsymbol{K_k} \cdot (\boldsymbol{\zeta_k} - \widetilde{\boldsymbol{C}} \cdot \hat{\boldsymbol{\xi}}_{\boldsymbol{k|k-1}}) \qquad (22)$$

$$\boldsymbol{P_{k|k}} = (\boldsymbol{I} - \boldsymbol{K_k} \cdot \widetilde{\boldsymbol{C}}) \cdot \boldsymbol{P_{k|k-1}} \qquad (23)$$

where $\boldsymbol{W}$ and $\boldsymbol{V}$ are the process and measurement noise covariance matrices, respectively, $\boldsymbol{K_k}$ is the *Kalman gain*, $\boldsymbol{P_{k|k}}$ and $\boldsymbol{P_{k|k-1}}$ are the covariance error matrix at time t_k and the prediction of the same matrix at time t_{k-1}, respectively, and $\hat{\boldsymbol{\xi}}_{\boldsymbol{k|k}}$ and $\hat{\boldsymbol{\xi}}_{\boldsymbol{k|k-1}}$ are the posterior and the prior estimation of the expanded state variables, respectively.

Note that the process noise covariance matrix $\boldsymbol{W}$ also takes into account the model's uncertainties, thereby allowing the state estimation even if the knowledge of the grid parameters is not perfect. Therefore, this iterative algorithm provides an accurate monitoring of both disturbances and state variables.

V. CONCLUSIONS

The state of a hybrid AC/DC μG must be known in order to perform optimal control of the system. However, economic issues make it unreasonable to monitor all of the state variables, so it is important to provide tools for the DHSE of the μG even when some of the desired measurements are not available.

According to this scenario, this paper provided of two methods for optimal placement of the MUs in hybrid AC/DC μGs, both of which have been used previously in AC networks. Specifically, these two methods are the minimum condition number method and the integer linear programming method. Then, the measurements collected by the MUs placed by both methods were used as inputs to the KF-based DHSE.

Only the theoretical aspects of the problem are reported in this paper, but the corresponding numerical applications are provided in the companion paper [29]. The companion paper compares the two methods for the optimal placement of MUs in terms of: (i) number of required measurements to guarantee the observability of the system; (ii) accuracy of the corresponding KF-based DHSE; and (iii) computational burden.

ACKNOWLEDGMENTS

This work was supported by Ministero dell'Istruzione dell'Università e della Ricerca (MIUR) in the frame of PON03PE_00178_1: Microgrid Ibride in Corrente Continua e

Corrente Alternata (MICCA). The authors also thank Prof. Guido Carpinelli for his constructive support in the realization of this work.

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