

# Reliability analysis of complex limit states of floating wind turbines

## 漂浮风力涡轮机复杂极限状态的可靠性分析

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**Abstract** - Offshore wind turbines are developing at a rapid pace and deployments are moving to deeper waters constituting floating support structures as a feasible option both technically and economically at depths that exceed 50 m. Experience of more than 50 years from the Oil & Gas industry has provided structural configurations and established methodologies and standards for the design of floating support structures with varying level of applicability to offshore wind applications, however, the different nature of loading that include a significant operational cyclic loading in addition to the environmental loads, the fact that those structures are designed for volume manufacturing and their limited consequences in the case of failure suggest a probabilistic approach to design and analysis as a pertinent practice towards cost reduction of capital expenditure and operational management. This paper presents a systematic methodology for reliability analysis of the floating support structures, focusing on the case of the analytical derivation of a fundamental limit state for stability under stochastic model inputs that is able to predict very small probabilities of failure.

**Keywords** –Offshore wind turbines, floating support structures, reliability analysis, limit states

### I. INTRODUCTION

With the need to increase renewable energy's share in global energy production and to exploit offshore wind resources, wind farms are moving further and further offshore into deeper waters. In water depths greater than 50 meters, bottom-mounted (i.e. fixed) support structures for offshore wind turbines do not remain the most economically viable option [1]. A transition from fixed to floating support structures is essential for deep offshore wind farms to become economically viable in the near future.

Whilst it is beneficial to utilize experience from the oil & gas industry during the design and manufacturing of floating wind turbines, the different nature of loading that include a significant operational cyclic loading in addition to the environmental loads, and the fact that those structures are

designed for volume manufacturing and their limited consequences in the case of failure suggest a probabilistic approach to design and analysis as a pertinent practice towards cost reduction of capital expenditure and operational management. In addition to that, the harsh offshore environments are characterized by highly stochastic variables which should be systematically incorporated to the design process in order to avoid accumulation of unnecessary conservatism which ultimately increases total cost.

This paper presents a systematic methodology for reliability analysis of floating support structures, focusing on the case of the analytical derivation of a fundamental limit state for stability under stochastic model inputs able to predict very small probabilities of failure. A sensitivity analysis of the solution based on First Order Reliability Methods (FORM) as well as variation of the statistical properties of the variables that are modelled stochastically, illustrates the performance of the limit state derived for probabilistic analysis. Applicability of the methodology can be extended to other limit states, such as mooring line design and incorporation of coupled dynamics of the complex system as well as inform the requirements for inspection, maintenance, or operational control based on the current state of the structural system.

### II. FLOATING WIND TURBINES

The trend so far has been to 'marinise' the optimal onshore configuration (that is, the 3-bladed horizontal axis wind turbine (HAWT)) for use in floating offshore applications. The operating environments found in onshore and floating offshore applications are significantly different, and hence the optimal wind turbine configuration may not be the same for both cases. An alternative to the HAWT that may be more suited to floating applications is the vertical axis wind turbine (VAWT). Although it may have lower individual turbine power coefficients, the generator and transmission machinery is found at the base of the turbine (rather than at the top of the tower for the HAWT) resulting in a lower centre of gravity.

Furthermore a VAWT generates smaller thrust forces and overturning moments than a HAWT, resulting in a smaller support structure being required as compared to a similar-sized HAWT [2].

Following on from the oil & gas industry, the three main types of floating support structures envisaged for floating wind turbines are semi-submersible, spar and tension-leg-platform that achieve stability mainly through buoyancy/waterplane area, ballasting and mooring lines, respectively. In the pursuit of reducing capital and operating costs, a number of concepts have been proposed that are a hybrid of some of the above mentioned platforms, for example, the tension-leg-buoy that combines a spar with a taut mooring system. Likewise a semi-submersible could also have a hybrid slack and taut mooring system to maximize the advantages of each type.

In this paper two floating VAWTs shall be considered, utilizing the spar and semi-submersible floating support structures as presented by Borg and Collu [3], and depicted in Fig. 1.

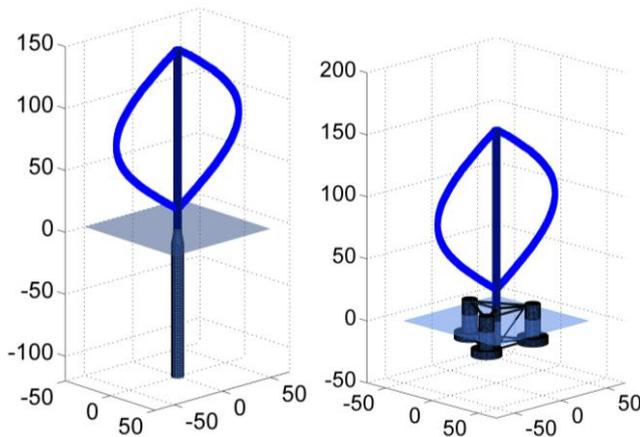


Fig.1: Left – spar-type floating VAWT; Right – Semi-submersible floating VAWT.

### III. CAPSIZING LIMIT STATE FOR FLOATING WIND TURBINES

During the preliminary design of the floating support structure, one of the design drivers is the system restoring stiffness in the pitch to counteract the pitch overturning moment generated by the wind turbine. For a moored floating structure, this restoring stiffness is a combination of hydrostatic and mooring system stiffness in pitch [4]:

$$C_{55} = C_{55}^h + C_{55}^m$$

The pitch hydrostatic stiffness is directly related to the metacentric height of the floating structure, and is given by the following equation [5]:

$$C_{55}^h = \rho g V (GM_L)$$

where  $\rho$  is the fluid density,  $g$  is the acceleration due to gravity,  $V$  is the displaced volume of fluid by the structure, and  $GM_L$  is the longitudinal metacentric height which is given by:

$$(GM_L) = (KB) + (BM_L) - (KG)$$

where  $KG$  is the distance of the centre of gravity from the bottom of the structure,  $KB$  is the center of buoyancy from the bottom of the structure, and  $BM_L$  is the distance between the center of buoyancy and metacenter, and is given by:

$$(BM_L) = \frac{I_L}{V} = \frac{\iint x^2 dA}{V}$$

where  $I_L$  is the second moment of area of the waterplane area of the floating platform. The mooring pitch stiffness is obtained as a product of the surge mooring stiffness and the moment arm of the mooring surge line of action to the center of flotation.

The capsizing limit state can be established by identifying the maximum allowable pitch displacement of the floating wind turbine,  $\xi_5$ , such that the minimum required pitch restoring stiffness,  $C_{55,min}$ , is identified based on the maximum excitation moment,  $F_5$  [4]:

$$C_{55,min} = \frac{F_5}{\xi_5}$$

Thus the limit state function is defined as:

$$C(X) = C_{55} - C_{55,min}$$

and the zones defined by the limit state function are:

$C(X) > 0$	Failure Region
$C(X) < 0$	Safe Region
$C(X) = 0$	Critical Region

### IV. CONCEPTS OF RELIABILITY & PROBABILISTIC ANALYSIS

A reliability analysis of the structural designs is a systematic approach that allows evaluating the levels of safety and serviceability of the structure subjected to the uncertain input. Recently, such methodology has been established as an essential tool in analysis of the actual performance of the structures. Additionally, it formed the basic background for the structures design standards [6].

To determine the structure operability limits, this methodology assumes that the reliability of structures can be estimated based on the limit state function which captures the performance of the structure under loading. A condition under which the structure or its component does not satisfy its design requirements is called a limit state [7]. Each limit state can be characterized by  $n$  structure variables,  $X_i$ , which affects the structure response. A stochastic representation of these variables needs to be determined. Hence, following the mathematical notation, the limit state can be described as:

$$Z = g(X_1, X_2, \dots, X_n)$$

The critical value for the limit state, which distinguishes the safe and the failure region, is defined as:

$$Z = 0$$

Using the full distributional integration, the probability of failure can be computed within the integration limit of  $Z < 0$  using the following equation:

$$P_f = \int \dots \int_{g(x_1, x_2, \dots, x_n) < 0} f_x(x_1, x_2, \dots, x_n) dx_1 dx_2 \dots dx_n$$

where  $f_x(x_1, x_2, \dots, x_n)$  is the joint probability density function of the random variables  $(X_1, X_2, \dots, X_n)$ . As its accurate estimation requires a complex procedure, indirect methods, such as Monte Carlo simulation, are frequently applied. In the reliability analysis, the probability of failure  $P_f$  is often represented in terms of the reliability index  $\beta$ :

$$P_f = 1 - \Phi(\beta)$$

Where the notion  $\Phi$  is the inverse cumulative distribution function of the normally distributed reliability index.

The integration can be simplified by linearization of the limit state functions using Taylor series expressions. In First Order Reliability Methods (FORM), such as Hasofer-Lind method [8], which use first order Taylor series expression, the reliability index is approximated geometrically in an iterative process as the shortest distance between the limit state surface and zero point of the normalized U-dimensional space. This method, however, would produce inaccurate results if the limit state function is non-linear or has multiple minimal distance points. To enhance the accuracy and reliability of the prediction, the Second Order Reliability Methods (SORM), which use second order Taylor series expression, are also used. In this method, the reliability index is the shortest distance between the limit state function and an asymptotic curve rather than a straight line [9].

### V. NUMERICAL ANALYSIS: A CASE STUDY

Table 1: VAWTs design characteristics.

Parameter	Spar-type VAWT	Semi-submersible VAWT
Pitch mooring stiffness (Nm/rad)	311100000	87300000
Pitch hydrostatic stiffness (Nm/rad)	1.0447e+09	7.8780e+08
Buoyancy force (N)	80736300	139939650
Draft (m)	120	20
Mass (tn)	8125.2	14108
Centre of buoyancy (m)	57.863	6.813
Centre of gravity (m)	45.37	11.07
Outer radius (m)	4.8	3
Inner radius (m)	0	0
Cut-off wind speed (m/s)	25	25
Extreme wind speed (m/s)	45.1	45.1
Density (kg/m <sup>3</sup> )	1025	1025

Having defined a general form of the limit state functions, these are used to assess the probability that the system is not capable of restoring stiffness in pitch to counteract the pitch

overturning moment generated by the wind turbine. As mentioned above, two floating VAWTs, the characteristic of which is presented in Table 1, are considered.

In this analysis, three variables (pitch mooring stiffness, extreme wind speed and density) are treated stochastically. Due to uncertain nature of wind and wave loads, the coefficient of variation (COV) for wind speed and the pitch mooring stiffness is assumed to be 30% and 50 %, respectively. Conversely, the COV for the water density is assumed to be 10 %. For the sake of this analysis, it is assumed that these variables are normally distributed. The excitation moment has been calculated based on the VAWT characteristics, which is the same for both structures, and is expressed as a second order polynomial function of the wind speed as:

$$F_5 = 4077v^2 + 2.218 \cdot 10^{-9}v - 4.182 \cdot 10^{-8}$$

Moreover, the maximum allowable pitch displacement for the considered VAWTs is assumed to be 10°.

The reliability index was estimated using the iterative Hasofer-Lind method, which is one of the FORM methods.

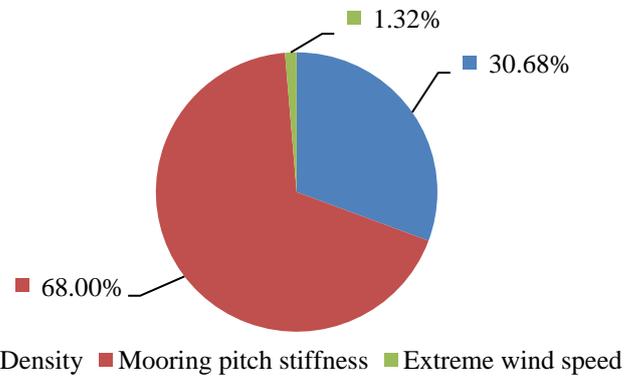


Fig. 2: Importance factors of the stochastic variables in estimation of the reliability index for the spar-type floating VAWT.

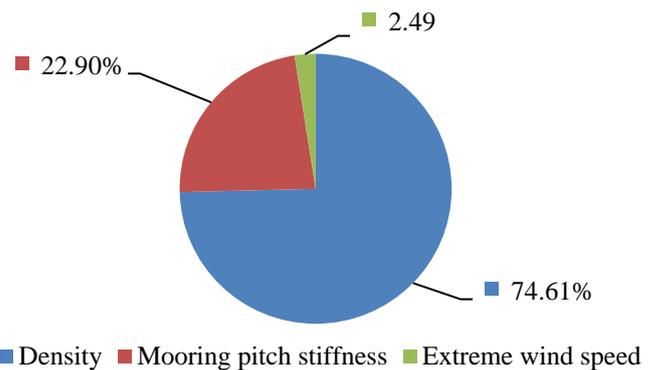


Fig. 3: Importance factors of the stochastic variables in estimation of the reliability index for the semi-submersible floating VAWT.

From Fig. 2 it is clear that in the spar-type floating VAWT the variation in the pitch mooring stiffness has the highest effect on the reliability index, right after the water density. Conversely, as shown in Fig. 3, the reliability index for the semi-submersible floating VAWT is mostly affected by the

water density. This means that the effect of the wave loads on the mooring pitch stiffness is partially avoided for the semi-submersible floating VAWT. However, this kind of VAWT is more prone to be affected by the wind loads, what is indicated by higher contribution of the extreme wind speed in the reliability index compared to the spar-type floating VAWT.

Table 2: Reliability analysis results.

Parameter	Spar-type VAWT	Semi-submersible VAWT
Reliability index $\beta$	6.92	8.95
Probability of failure $P_f$	2.24e-12	0

The results of the conducted analysis presented in Table 2 indicate that the reliability index for both structures is high. This means that the probability of the structure overturning due to variation in the wind and wave loads, as well as water density, is negligible. The analysis also showed that the semi-submersible floating VAWT would provide better performance in terms of system reliability. This probably results from the lower impact of the mooring pitch stiffness on the reliability index as identified above.

## VI. CONCLUSION

This paper presents a methodology for the reliability analysis of two kinds of floating support structures for VAWTs. The VAWTs background study is followed by a description and derivation of the capsizing limit state for the floating wind turbines. Next, the stability check is conducted through estimation of the reliability indices, and thus the failure probabilities, for the spar-type and the semi-submersible floating VAWTs. The analysis showed that the impact of the mooring pitch stiffness on the reliability of the structures is reduced for the semi-submersible floating VAWTs, at an expense of the wind speed. Nevertheless, application of such structure would result in increased reliability index, hence in lower probability of the structure overturning. Finally, a simplified model for coupling of the aero-hydro-servo-elastic induced dynamics of a VAWT and its effect on reliability estimation has been proposed.

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## NOMENCLATURE

$BM_L$	Distance between the center of buoyancy and metacenter
$C_{55}$	System restoring in pitch
$C_{55}^h$	System hydrodynamic pitch stiffness
$C_{55}^m$	System mooring pitch stiffness
$F_5$	Steady state excitation moment
$g$	Gravitational acceleration
$GM_L$	Longitudinal metacentric height
$I_L$	Second moment of area of the waterplane area of the floating platform
$KB$	Centre of buoyancy from the bottom of the structure
$KG$	Distance of the center of gravity from the bottom of the structure
$P_f$	Probability of failure
$v$	Wind speed
$V$	Fluid volume displaced by the structure
$X_i$	Random variables
$Z$	Limit state function
$\beta$	Reliability index
$\zeta_5$	System steady state pitch
$\rho$	Density